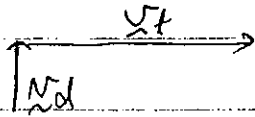


Q1



$$v_d = 12 \text{ m s}^{-1}$$

$$v_t = 40 \text{ km/h} = 40 \times 10^3 / (60 \times 60) = 11 \text{ m s}^{-1}$$

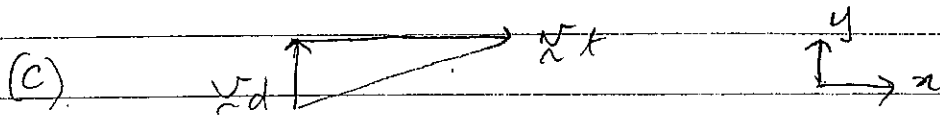
$$(a) \quad z - z_0 = -\frac{1}{2} g t^2$$

$$|\Delta z = -0.25 \text{ m}$$

$$\Rightarrow t = \sqrt{\frac{2(z_0 - z)}{g}}$$

$$= \sqrt{\frac{2 \cdot 0.25}{9.8}} = 0.226 \text{ s}$$

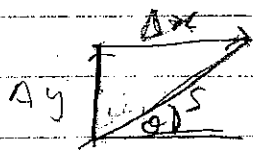
$$(b) \quad y - y_0 = v_{y0} t = 12 \times 0.226 = 2.7 \text{ m}$$



$$\underline{v} = \underline{v}_d + \underline{v}_t = v_d \underline{j} + v_t \underline{i} = (12 \underline{j} + 11 \underline{i}) \text{ m s}^{-1}$$

$$(d) \quad \Delta x = v_x t = 11 \times 0.226 = 2.49 \text{ m}$$

$$\Delta y = v_y t = 12 \times 0.226 = 2.71 \text{ m}$$



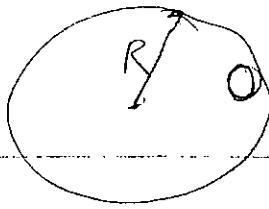
$$s = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{2.49^2 + 2.71^2} = 3.68 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{2.71}{2.49}\right) = 47.14^\circ$$

$\Rightarrow s = 3.7 \text{ m}$ at an angle 47° from the x -axis

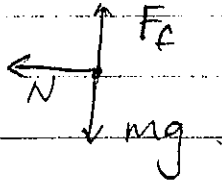
e) Yes, the flight time will be different.

Because of the horizontal acceleration, the dart will land (in general) in a different horizontal position to that found earlier. The different flight time will give a different vertical position.



2

(a)



(b)

$$\sum F_z = 0$$

(c)

$$F_f - mg = 0 \Rightarrow F_f = mg$$

$$F_f = \mu_s N \quad - \text{max. frictional force, on verge of slipping.}$$
$$\Rightarrow \mu_s N = mg$$

$$\mu_s = mg/N$$

(d)

$$\sum F_r = N = mv^2/R$$

(e)

$$\mu_s = mg/N = mg/(mv^2/R) \cdot R = Rg/v^2$$

$$\Rightarrow N = \sqrt{Rg/\mu_s}$$

(f)

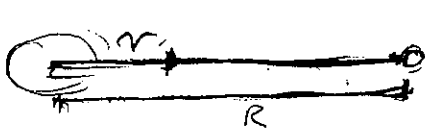
$$N = 7.0 \text{ m/s}^2$$

$$\Rightarrow \mu_s = Rg/v^2 = 2 \times 9.8 / 7^2 = 0.40$$

(g)

If μ_s were larger, person would still be supported.

If μ_s were smaller, person would fall.



3. (a)

$$F_{\text{tot}} = F_e + F_m =$$

$$= -\frac{GmM_e}{r^2} + \frac{GmM_m}{(R-r)^2} \quad \text{in direction of moon.}$$

(b)

$$\frac{GmM_e}{(R-r_0)^2} = \frac{GmM_m}{(R-r_0)^2}$$

$$\Rightarrow \frac{(R-r_0)^2}{r_0^2} = \frac{M_m}{M_e}$$

$$\left(\frac{R}{r_0} - 1\right) = \sqrt{\frac{M_m}{M_e}}$$

$$R/r_0 = \sqrt{\frac{M_m}{M_e}} + 1.$$

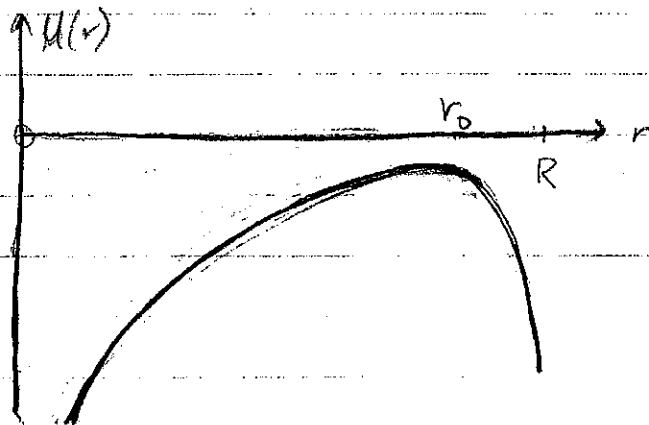
$$\Rightarrow r_0 = \left(\sqrt{\frac{M_m}{M_e}} + 1\right)^{-1} R$$

$$= \left(\frac{1}{9} + 1\right)^{-1} R$$

$$= \frac{9}{10} R.$$

(c) $U(r) = -\frac{GmM_e}{r} - \frac{GmM_m}{(R-r)}$

(d)



(e) $U(r_0) = -\frac{GmM_e}{r_0} - \frac{GmM_m}{(R-r_0)} = -Gm \left(\frac{M_e}{\frac{9}{10}R} + \frac{M_m}{\frac{1}{10}R} \right)$

$$= -\frac{Gm}{R} \left(\frac{10}{9} M_e + 10 M_m \right)$$

$$= -\frac{Gm}{R} M_e \left(\frac{10}{9} + 10 \cdot \frac{M_m}{M_e} \right)$$

$$= -\frac{GmM_e}{R} \left(\frac{10}{9} + 10 \cdot \frac{1}{81} \right) = -1.3$$

(e) cont.

$$\Rightarrow U(r_0) = -1.23 \frac{GmM_e}{R}$$

(f) Minimum energy to reach moon:

$$E = U(r_0) = - \frac{GmM_e}{R} (1.23)$$

$$(g) E = -1.23 \frac{GmM_e}{R} = \frac{1}{2} m v_0^2 - \frac{GmM_e}{R_e}$$

$$\begin{aligned} \Rightarrow \frac{1}{2} m v_0^2 &= \frac{GmM_e}{R_e} \left(-1.23 \frac{R_e}{R} + 1 \right) \\ &= \frac{GmM_e}{R_e} \cdot 0.9795 \end{aligned}$$

$$\Rightarrow v_0 = \sqrt{\frac{2GmM_e}{R_e}} \cdot 0.99$$

4.

$$\begin{array}{cc} m_1 & m_2 \\ \circ_1 & \circ_2 \\ x_1 & x_2 \end{array}$$

$$(a) \quad x_{cm} = \frac{1}{m_1 + m_2} (m_1 x_1 + m_2 x_2)$$

$$(b) \quad v_{cm} = \frac{dx_{cm}}{dt} = \frac{1}{m_1 + m_2} (m_1 v_1 + m_2 v_2)$$

(c) mom. cons.

$$m_1 u_1 = m_1 v_1 + m_2 v_2 \quad (1)$$

Energy cons.

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad (2)$$

$$(1) : \quad m_1 (u_1 - v_1) = m_2 v_2$$

$$(2) \quad m_1 (u_1^2 - v_1^2) = m_2 v_2^2$$

(2)/(1).

$$\frac{(u_1 - v_1)(u_1 + v_1)}{u_1 - v_1} = v_2$$

$$v_2 = u_1 + v_1$$

$$(1) \quad m_1 u_1 = m_1 v_1 + m_2 (u_1 + v_1) = (m_1 + m_2) v_1 + m_2 u_1$$

$$-(m_1 - m_2) u_1 = (m_1 + m_2) v_1$$

$$\Rightarrow v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$$

$$\begin{aligned} v_2 &= v_1 + u_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} + \frac{m_1 + m_2}{m_1 + m_2} \right) u_1 \\ &= \frac{2m_1}{m_1 + m_2} u_1 \end{aligned}$$

$$(d) \quad v_{cm} = \frac{1}{m_1 + m_2} (m_1 u)$$

$$(e) \quad v_{cm} = \frac{1}{m_1 + m_2} \left(m_1 \frac{m_1 - m_2}{m_1 + m_2} + m_2 \frac{2m_1}{m_1 + m_2} \right) u$$

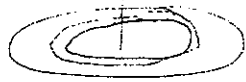
$$= \frac{1}{(m_1 + m_2)^2} (m_1^2 + 2m_1 m_2) u$$

$$= \frac{m_1 (m_1 + m_2)}{(m_1 + m_2)^2} u$$

$$= \frac{m_1 u}{m_1 + m_2}$$

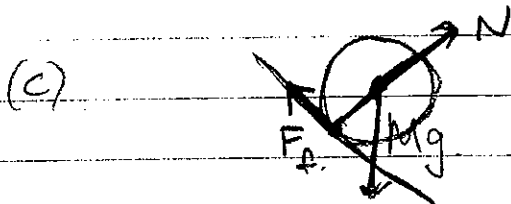
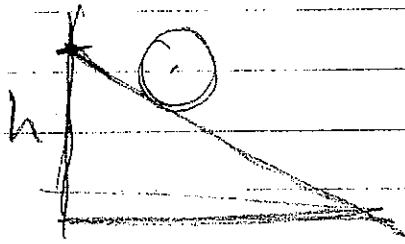
(f) Yes, (d) & (e) agree. There are no external forces acting on the system,

$$F_{ext} = \frac{dP}{dt} = 0 \Rightarrow P \equiv \sum_i p_i = M v_{cm} \text{ is constant}$$



5. (a) $dm = \sigma dA = \frac{M}{\pi R^2} 2\pi r dr = \frac{2M}{R^2} r dr$

(b) $I = \int_0^R r^2 dm = \frac{2M}{R^2} \int_0^R r^3 dr = \frac{2M}{R^2} \cdot \frac{R^4}{4} = \frac{MR^2}{2}$



The frictional force produces a torque about the CM.

(d) $v_{cm} = R\omega$

(e) $Mgh = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$

$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} \frac{MR^2}{2} \cdot \left(\frac{v_{cm}}{R}\right)^2$$

$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} M \frac{R^2}{2} \frac{v_{cm}^2}{R^2}$$

$$\Rightarrow Mgh = \frac{3}{4} M v_{cm}^2$$

$$\Rightarrow v_{cm} = \sqrt{\frac{4}{3} gh}$$

(f) $Mgh = \frac{1}{2} M v_{cm}^2 \Rightarrow v_{cm} = \sqrt{2gh}$

(g) When the disc rolls without slipping all the energy is converted to translational kinetic energy & so it is faster than when the disc rolls & some of the energy is used for rot. motion.